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LETTER TO THE EDITOR

Exotic gauge potential representations and ghost counting via orthosymplectic BRS supersymmetry

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Abstract. Generalised gauge fields and their accompanying ghosts are naturally accommodated in representations of an orthosymplectic supergroup. Counting rules are given for arbitrary irreducible tensors of $OSp(n/m)$, and the connection with representations of the transverse orthogonal group $O(n-m)$ for 'massless' states is pointed out.

A recent series of articles (Townsend 1980, Siegel 1980, Thierry-Mieg 1980, Duff 1981, Obukhov 1982) on antisymmetric tensor gauge potentials contains discussion of the quantisation problems associated with these exotic fields and the attendant ghost structures (Nielsen 1978, Kallosh 1978, Namazie and Storey 1979). The last paper, by Obukhov, clarifies the role of residual gauge symmetries as the families of ghosts are sequentially introduced and gives a proper count, based on the Fadeev Popov ansatz, of the number of degrees of freedom. Further, the BRS transformations of these families are introduced and the functional BRS gauge identity is exposed. See also Marchetti and Tonin (1981).

In this note we would like to exhibit the counting argument on the basis of orthosymplectic BRS supersymmetry. We shall frame our arguments for the most general case of an m-extended supersymmetry attached to a space–time of n dimensions, so that fields are placed in representations of the tangent space supergroup $OSp(n/m)$. In practice $n=4, m=2$ is the most pertinent physical example, but supergravity practitioners are interested in n values which go up to 11. In an earlier paper (Delbourgo and Jarvis 1982) we suggested that the representations of the orthosymplectic superalgebra offered the simplest and most lucid way of arriving at the collection of required gauge plus ghost fields and we gave a few illustrations. Here we will give the complete counting rules for the most general exotic potential representation.

Let ϕ_R represent a vector of $OSp(n/m)$. It can be decomposed under $O(n)\times Sp(m)$ into ϕ_ρ and ϕ_r which are the basic vectors of the space–time and BRS symmetry groups (for the usual extended BRS field variations m equals two). Consider first the case of a fully 'antisymmetric' rank p generalised gauge field, $\phi_{[R_1\ldots R_p]}$. It decomposes into the set

$$\phi_{[\rho_1\ldots\rho_p]},\ \phi_{r_1[\rho_2\ldots\rho_p]},\ \phi_{(r_1r_2)[\rho_3\ldots\rho_p]},\ \ldots,\ \phi_{(r_1\ldots r_p)}$$

i.e. into

$$\sum_{k=0}^{p}\oplus\phi_{(r_1\ldots r_k)[\rho_{k+1}\ldots\rho_p]}. \qquad (1a)$$

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The 'ghost' field with k Sp(m) components and $p-k$ O(n) components has dimension $\binom{m+k-1}{k}\binom{n}{p-k}$. (It is an a-number field when k is odd.) The *number of physical components N* of the OSp(n/m) superfield is its *graded dimension*. Thus

$$N=\sum_{k=0}^{p}(-1)^k\binom{m+k-1}{k}\binom{n}{p-k}=\binom{n-m}{p}, \tag{2a}$$

where we have used the basic summation formula (Edmonds 1957)

$$\sum_k \frac{(A+k)!\,(B-k)!}{(C+k)!\,(D-k)!}=\frac{(A+B+1)!\,(A-C)!\,(B-D)!}{(C+D)!\,(A+B-C-D+1)!} \tag{3}$$

in one of its guises. When $m=2$, ($2a$) agrees with earlier work and in particular with Obukhov's unexceptionable formulation. Observe that the result equals the dimension of a p-fold antisymmetric tensor in $(n-m)$ dimensions, thereby substantiating Curtwright's (1982) view that this field should represent a massless object with the little group $O(n-2)\times T_{n-2}$, in which the translations are trivially represented (gauge invariance).

Next consider the 'symmetric' rank p generalised gauge field $\phi_{(R_1\ldots R_p)}$ which can be broken up into

$$\sum_{k=0}^{p}\oplus\phi_{[r_1\ldots r_k](\rho_{k+1}\ldots\rho_p)}.$$

Because of the supertracelessness we have to impose several conditions

$$\eta^{\rho\sigma}\phi_{(\rho\sigma\ldots)[tu\ldots]}-\eta^{rs}\phi_{[rs\ldots](\tau v\ldots)}=\eta^{RS}\phi_{(RS\ldots)}=0 \tag{1s}$$

on the superfield components. Now because

$$\dim[\phi_{[r_1\ldots r_k](\rho_{k+1}\ldots\rho)}]=\binom{m}{m-k}\binom{n+p-k}{p-k}, \tag{4}$$

the overall graded dimension, before applying constraints, and using (3), is

$$N'=\sum_{k=0}^{p}(-1)^k\binom{m}{m-k}\binom{n+p-k}{p-k}=\binom{n+p-m}{p}. \tag{5}$$

Therefore, the physical number of degrees of freedom, after applying the constraints—which amounts to subtracting the degrees of freedom of the contracted field $\eta^{R_1R_2}\phi_{(R_1R_2R_3\ldots R_p)}$—will be given by

$$N=\binom{n+p-m}{p}-\binom{n+p-m-2}{p-2}. \tag{2s}$$

For $m=2$ this is precisely the number of components of a symmetric rank p (trivial) representation of the Euclidean group in $n-2$ dimensions, again in agreement with Curtright's count.

These conclusions apply equally well to any irreducible tensor representation† of OSp(n/m), not just the symmetric and antisymmetric cases. This follows directly from Bars and Balantekin's (1981) construction of supergroup characters. An alternative demonstration, more closely related to the above tensor methods (Dondi and Jarvis 1980, 1981), starts from the branching rule $OSp(n/m)\supset O(n)\times Sp(m)$ for a general

† For simplicity we avoid indecomposable tensors (atypical representations) in this discussion.

supertraceless irreducible tensor representation (corresponding to Young diagram λ):

$$[\lambda]\downarrow\sum_{\zeta}[\lambda/\zeta]\times\langle\tilde{\zeta}/B\rangle. \qquad (6s)$$

Here $[\alpha]\times\langle\tilde{\beta}\rangle$ labels an irreducible representation of $O(n)\times Sp(m)$, $\tilde{\beta}$ is the transpose of β, the summation is over all divisible diagrams ζ, and the final factor involves division by all admissible diagrams in the series $\{\{0\},\{1^2\},\{1^4\},\ldots;\{2^2\},\{2^21^2\},\ldots;\ldots\}$ (see Black *et al* 1982, King 1982).

The connection of (6s) with representations of the ordinary Lie algebra $O(n-m)$ follows from the observation that formally one can express characters of the latter via the chain

$$O(n-m)\subset U(n-m)\subset U(n)\times U(m)\supset O(n)\times Sp(m)$$

provided the class parameters are suitably restricted. Thus

$$[1]\uparrow_r\{1\}\uparrow_r\{1\}\times\{0\}-\{0\}\times\{1\}\downarrow_r[1]\times\langle 0\rangle-[0]\times\langle 1\rangle,$$

for the fundamental representation, will be recognised as the gauge and ghost field assignment for a rank one tensor. The result for a general tensor may be obtained from the algebra of plethysms (see e.g. Wybourne 1970) and the S-function calculus:

$$[\lambda]\downarrow_r\Sigma(-1)^{|\zeta|}[\lambda/\zeta]\times\langle\tilde{\zeta}/B\rangle. \qquad (6)$$

Evaluating the identity, (6) gives $O(n-m)$ dimensions in terms of $O(n)\times Sp(m)$ dimensions: precisely the same expression results from (6s) for the graded dimension, where the sign factor $(-1)^{|\zeta|}$ signifies an a-number ghost field whenever ζ has an odd number of boxes. Hence, we have established that the number of physical degrees of freedom of a quantised gauge potential assigned to an arbitrary representation of $OSp(n/m)$ equals the number of components of the corresponding representation of $O(n-m)$.

The extended BRST transformations will connect the various fields in the $OSp(n/2)$ supermultiplet in such a way that successive Taylor series coefficients in the superfield expansion

$$\phi_{Ri}(x,\theta)=A_{Ri}(x)+\theta^m B_{rim}(x)+\tfrac{1}{2}\theta^2 C_{Ri}(x)$$

are BRST transforms of one another. These and other superfield aspects (couplings, superactions, dimensional reduction methods, etc) will be explored elsewhere; this letter is merely concerned with demonstrating the mechanical simplicity of the orthosymplectic framework for arriving at the ghostly retinue of exotic gauge fields.

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